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A Fresh Look at an Old Problem: Saturation in the Retail Market and How it Affects Both Retailers and Consumers

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Abstract

Objective: Traditional saturation analysis on competitive location decision science focuses on diminishing returns for incumbents and newcomers in a specific spatial location pertaining to commercial retail potential past a certain point of market saturation. **Methods/Findings**: This study looks at this problem but employs a different approach to the subject altogether, wherein saturation is no longer a variable affecting only retailers but one that affects both: the marginal utility of consumers and the revenue of retailers albeit differently. A new mathematical model is proposed based on selected papers, contributing new insight into an already widely discussed subject. **Application:** Analysis shows that it is important for competitive location decision-making to address saturation from both sides of the overall competitive location decision issue, not just from the retailers' standpoint.

Keywords: Clustering, Competitive Location, Market Share, Retail Industry, Retail Saturation Index

1. Introduction

The concept of saturation is not new[1], when explaining his "Theory of Evolution", discussed saturation as a factor affecting the evolution of species. According to the saturation thesis, that is relevant not only to humans but to all living organisms that seek to accommodate themselves in their environment as dynamic systems, the species could not increase substantially beyond the original number without jeopardizing the whole population. In the long term however, the species could increase at the expense of other related (extinct) species. Hence an important parallel can be made with the retail facility location problem. Competition in all its forms and phases is the major factor at play and competing on location becomes all too relevant when saturation effects consider retailers and consumers. Thus saturation should be factored into the equation of competitive location decision-making analysis yet its management and analytical approach will differ based on whether one focuses on the retailers' (incumbent and newcomers) or the consumers' standpoint. Until now factors such as facility attractiveness based on location distance to the consumers and square meters of floor space available.

Thus saturation is viewed here as an equally important issue if not more so than traditional factors such as proximity to urban areas, minimizing transportation costs and the minimization of the weighted sum of distances from a given set of point sites. All these issues can also apply to cluster analysis and are widely addressed by the literature[2–6].

Surprisingly, saturation appears as the missing link in the overall analytic framework on the subject when the effects on the consumer are taken into account and as such

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it plays an important role that cannot be dismf7issed. This fact has somehow been underestimated, overlooked and minimally understood thus far when analyzing the competitive location decision-making analysis holistically. Saturation takes place when a number of retailers already present in a specific commercial area has reached a certain point where the level of competition among them is very high. To outsiders, of whom the customer is an authorized voice, the number of stores or shops appears to be excessive compared to the average number of consumers visiting the area on a regular basis. Thus the area is said to be saturated with vendors, surpassing the level of healthy competition that can benefit consumers and entering a red zone a red ocean if you will–where most players try to eat each other out desperately. In such scenarios it is not infrequent or uncommon to see that although the number of players already operating is quite high and competition is fierce, incumbents continue to face new retailers coming in to establish themselves in the same commercial location with detriment for all following the law of diminishing returns. Similar cases can be observed with marginal productivity in economics, when one factor of production or operations is increased while other factors are held relatively constant. In this case, the output per unit of the variable factor will eventually diminish and it will continue to do so until it reaches a point of negative return that is a red zone where the system simply loses money. There are several factors that may influence saturation of retail facilities in a given community, such as lack of adequate regulation in urban development, new changes imposed by municipal authorities and changing trends in urban development regulations. Also the massive migrations of people from one region or urban zone to another is an important factor as is the case in Chile, Mexico and many parts of the USA for example. Yet in the case of Chile's metropolitan region and the city of Santiago in particular, none is more prevalent and pervasive than the concentration of capital and commercial land. This problem is never more prevalent and omnipresent than in retail. The intuitive linkage between capital concentration and retail saturation can be directly associated with the capacity to buy commercial land or land with the prospect of becoming commercially viable in hope that urban planning trends favor them in the near future. Therefore, when the right time comes, they act fast and aggressively, opening new facilities everywhere they deem necessary to protect future encroachment of new competitors, thus intentionally saturating the market swiftly and quite craftily.

In[7] focused on the dynamics of population growth and their evolution affecting different regions of the United States employing nonlinear analysis. In their study they show that exact analytic solutions for population evolution trends along symmetry paths can be found and where dynamic stability analysis can point to which options of evolution are preferred by certain populations and how this leads to stable end configurations. A nonlinear model for the migration of one population between L primordially equivalent regions is introduced in their study, where the agglomeration trend k is the relevant system parameter. In[8] base their contribution on the index proposed for measuring the localization of economic activity. They develop an alternative localization measure that is consistent with the theoretical framework originally proposed by these authors, albeit not considering saturation as a variable that can play a role in location decision. In[9,10] propose a new spatial multivariate model to predict the number of new businesses at a county level in the state of Texas, considering relevant factors such as agglomeration, economies/diseconomies, industrial specialization indices, human capital, fiscal conditions, transportation infrastructure and land development characteristics. However, no mention is made of the role that saturation may also play when considered among others factors addressed by applied research. In regards to spatial economic concentration and spatial clustering in certain areas, state that while they may be useful and applicable they stem from calculations within individual areal units and thus are insensitive to the proximity of regions or to neighboring effects. Considering that commercial developments transcend areal units, economic clusters may well cross regional boundaries and may be divided into individual areas not representative of the real clustering phenomenon expansion that is taking place. Saturation may become quite relevant an issue whenever such spatial economic concentration and spatial clustering develop to extents unplanned and/or unexpected, thus its relevance for regional analysis. In[11,12] argue that many facility location decision models ignore the fact that for a facility to survive it needs a minimum demand level to cover costs. Saturation conditions have a direct effect on such minimum demand level for retailers. In their paper they present a decision model for a firm that wishes to enter a spatial market where several competitors are

already located. In that scenario for each outlet that sets in the area there is a demand threshold level that needs to be achieved in order for the retailer to survive. However, as with other pieces of literature, there is no concern for saturation levels affecting new entrants and how, due to these new entrants, some competitors will not be able to meet the minimum demand level threshold because of high saturation conditions already present in the area albeit unperceived, and therefore will disappear. As seen upon reviewing the literature, one finds that the concept of retail saturation has not been adequately accounted for in competitive facility location theory and in location science in general, thus remaining significantly underdeveloped. Therefore, this paper seeks to fill a void in the literature by choosing to introduce saturation as a qualitative factor to be dealt with as it has been done in the past when analyzing facility location decision-making. Saturation is a dynamic and widespread process, particularly in the retailing industry, which needs further attention. The phenomenon arises as a deterrent for new entrants and as a burden to incumbents. It is also a factor that can trigger cannibalism among competitors given the right set of conditions[13]. Nevertheless, as long as there is a market for a product, there will be new opportunities for distributors and stores to open despite saturation levels. Only recently and very marginally has the idea of saturation been brought up, rather implicitly in papers by[14–21] it is also addressed in an explicit but very limited manner by[22,23] in the economic, geographical and urban planning context; and it is also treated by[24–30] and but in a limited fashion. The concept of saturation in the retail industry relates to retail markets at every level, where if additional stores were to open for business, it would prove unsustainable for new entrants and would no doubt add unwanted complexity and chaos to the consumer. This can be observed very often in the form of product repetitiveness, price uniformity, tug of war between vendors and customers and increasing levels of entropy in the retail system in which the purchasing decision and facility choice processes take place, in the context of the available consumer demand. Likewise, excessive market concentration in overlapping markets may be a cause of concern for market regulators, while mergers of complementary (i.e. non-overlapping regions) are less likely to cause concern. Excessive market concentration is one of the main factors fueling retail saturation. A market is saturated when it can no longer support/sustain more retail activity and therefore the opening of new facilities proves fruitless. Weaker outlets close or sell out and consolidation of vendors takes place. The impact of government planning guidelines, severely restricting the potential for new retail sites would intuitively seem to add to the likelihood of retail saturation arriving sooner rather than later. What is certain though is that the more capital concentration there is, the easier it is for incumbents to secure locations and avoid the influx of new entrants. Retail stores location decision is vital for the retail industry in general. Competitive facility location models seek the best location(s) for one or more new competing facilities that maximize the market share captured by the new outlets or else to keep the market share of existing stores. In general terms, competition in facility location is defined simply as that situation where two or more firms compete in a given market space for one or more sites regardless of what the space representation choice may be for the modeler, and that there are no asymmetries of any kind that may incline the balance towards one or the other. In other words, they compete on equal terms. However, in the real world this premise does not hold as some have considerable advantages with respect to others, especially when the market space is mature, with many players already well established. Here again a difference of opinion emerges with respect to the traditional line of thought expressed by[31,32] in that, although there may be some type and degree of interaction among firms once a location decision is made and executed, the new location decision will not necessarily affect its competitors' market share. This is due to the fact that although the location decision had been expected to fulfill expectations of capturing a bigger market share for the firm, there may well be a significant level of saturation already present in such a market and therefore a new entrant will likely not affect other players or else the effect will be very marginal at best.

2. The Threshold Concept and how it Determines Retail Market Saturation

The idea of a threshold, which gave birth to the threshold model, is not recent and implicitly points to the role of saturation in the competitive facility location problem discussion. This concept is thoroughly discussed and the authors correctly point to the idea of a minimum threshold level to be attained (thus implying that an important level of market saturation already exists) in order to survive in a

highly competitive market such that for each outlet to be open there is a certain demand threshold level that has to be achieved, otherwise it is not economically viable to enter such a market. In another important paper the idea of a minimum threshold emerges again in similar manner, this time pertaining to the fact that services in general, need to have a minimum catchment area to be in business, implying again that there may be a certain degree of saturation already present in the market, so that the likelihood or rather the possibility of finding that there is still room for more facilities to open must be carefully evaluated. In this paper they present a model to locate the maximum number of services that can coexist in a given region without having losses, taking into account that they need a minimum catchment area to exist. On the other hand, also touched on the subject of reaching a threshold on the competitive location problem and again in a somewhat similar problem this time on a network is discussed. Alternatively, one can look at the other side of the coin where one finds competitive clustering which presents location strategies that appear to be counterintuitive compared with the traditional analytical models which abound in the competitive facility location literature. Excessive clustering and high degrees of retail saturation will ultimately impact profitability of the whole and prove detrimental in the long run even for the consumers.

3. Considering a New Approach in the Analysis of an Old Problem: When Saturation is Factored into the Customer Experience

Competitive facility location is rooted in the work of[33] who sought to determine the optimal location of two competing vendors on a line segment representing a predefined area (for example, two ice-cream vendors along a beach strip). However, his work was confined to only two players, who had plenty of access to move freely to a different location and at no cost. In the real world however competition levels in competitive facility location is a serious issue that is likely to determine the success or failure of a firm's investment decision. A firm must live with its facility planning and location decisions for a long time, particularly in manufacturing and also when long-term retailing contracts are present and these decisions affect several factors, such as operating efficiency, economy of scale, supply chain management, ease of scheduling, maintenance costs. It all comes down to profitability. Therefore, an adequate analysis of all relevant variables that pertain to the competitive facility location planning process needs to be done in an interesting paper titled "Retail market share and saturation", that focuses on market share and share of space of smaller size retailers competing in certain regions, addresses the negative effects of saturation in retailing, as he looks at the individual retailers and attempts to analyze the likelihood that firms gaining extra market share as a result of extra floor space in a particular city or region when faced with saturation. For a long time, saturation and particularly retail saturation has been confined to the turf of geographers and urban planners and has also been extensively discussed in economics, but only timidly in the last couple of decades has it been addressed by researchers in location science. A seminal paper, although very limited in scope with regard to saturation, was done. On the other hand, unlike in manufacturing and industrial environments, where a long history of community complementarities, cooperation and solidarity among different players is known to exist, especially in very dense areas where one sees one facility right next to the other for kilometers, like in large industrial parks and manufacturing clusters, the retail facility industry is highly uncooperative and extremely competitive. Candidly put it "the competition of retail trade is very furious". The only potential cooperation that can emerge amongst the different players is outright collusion. In a similar manner, when a market is already full or close to being full (crowded with competitors), even when complementarities and mutual benefits may result for and amongst the different players such as in competitive clustering (so as to somehow ease the competition level in certain sectors of the community) profitability is certainly decreasing for the whole, with each new player that wants to step into the market. Thus saturation, whatever its face and degree may be, has to be factored in and considered with foresight in all its complexity, with a deeper analytical stance whenever firms are in the process of choosing site locations for their new facilities. Indeed, what seemed to be a very mild threat at first may well turn out to be a disaster in terms of investment for not having done the saturation assessment research adequately? However, slight the saturation potential may be in a market, region or urban area, and regardless of the stage in which the firm may be, when analyzing a new retail facility location decision, it has to be incorporated into the equation and thoroughly analyzed.

4. The Index of Retail Saturation (IRS) and the Buying Power Index (BPI)

The Index of Retail Saturation (IRS) is not new but it is seldom elaborated beyond its definition with very few exceptions like. IRS is simply the ratio of demand for a product or service divided by the available supply in a given area, and is computed as follows: IRS = C∗RE RF where IRS is the index of retail saturation for an area; C is the number of households (potential customers based on statistical family members number) in the area. RE stands for annual retail expenditures for a particular line of trade per household in the area; and RF is the square footage of retail facilities of a particular line of trade in the area. IRS varies between 0 and 1 (or 0 and 100%). On the other hand, the Buying Power index (BPI) is an indicator of a market's overall business potential and it is comprised of weighted measures of effective buying income, retail sales and population size. In the United States, the formula is used as follows: BPI = 0.5 (the area's percentage of U.S. effective buying income) + 0.3 (the area's percentage of U.S. retail sales) + 0.2 (the area's percentage of U.S. population). "A Location Allocation Model for Facility Planning in a Competitive Environment" proposed to improve on the assessment of multiple locations in competitive environments by developing an approach to finding desirable locations based on the concept of centroids proposed by[34–38] which is based on the minimax strategies used in game theory. Quoting from their paper to get a better glimpse for their motivation, one reads: "The effect of competitive facilities on location performance is either ignored or assumed to be easily predictable. Ignoring the competitive environment might not be a drawback for public facility locations, but the application of location allocation models to many private sector problems requires us to consider competition explicitly. In a competitive environment a location deemed desirable initially may become undesirable as competitors locate additional facilities in order to achieve their own objectives. Agreeing with the latter statement, one would add that not only the competition should be considered explicitly, but saturation level as well. Saturation, in competitive facility location in general and in retailing in particular, results from an excessive degree of competition among retailers which results from an excessive number of these located in one particular zone. The situation deteriorates rapidly past a certain point as more and more facilities locate into the market area or urban community, wherein the situation reaches a point of no return meaning literately that a decreasing marginal utility for the whole occurs with each new facility that locates in the area. "Competition and regional constraint are two vital impact factors in retail stores' location decision" and add that in their review of the literature, only one factor is concerned. The authors also affirm that "the company which will locate new retail outlets should concern the consumptive capacity of the regions or zones. In order to prevent excessive competition, the government also establishes some trade location ordinances to restrict the number of retail outlets in the zones with IRS. However as true as this may be in some parts of the world, it is also very lax and extemporaneous in many others. Usually urban ordinances come late and often fall short in regulatory specificity thus failing to safeguard both consumers and potential newcomers effectively against the saturation potential of markets. They argue that with the known IRS of a particular location, the maximum number of facilities minus the existing facilities leaves the maximum number of new facilities max P that can still locate in the area. Little is said about saturation as a factor to be dealt with. They fail to add that companies wishing to locate new retail outlets should concern the IRS of the region or urban area where they hope to gain market share. Later on they add that "the new entering firm can obtain the reasonable number of stores to locate in this region based on IRS. Furthermore, based on their review of the subject literature, they argue that a basic assumption in spatial interaction models is that the utility function Uij for a customer at node i using a facility at node j can be written as a ratio of a non-decreasing function F of the attractiveness Aj assigned to the new facility, and a non-decreasing function H of the travel distance dij with utility function $Uij = F(Aj)$ $H(dij)$ (1). According to the authors, a typical form of Uij is $Uij = Aj\alpha dij\beta$ (2) where α, β are parameters reflecting customer sensitivity to attractiveness Aj and travel distance d. In the literature the values are taken as: α = 1 and β = 2. In addition, they remark that another popular form for Uij involves the use of exponential functions instead of the power function in (2). Likewise, regarding distance ij d and for the sake of accuracy, according to the authors, the market share estimate may be enhanced by replacing the square of distance dij 2 with dij 2 + εi 2 using ε 2 = R 2 2 so the distance expression becomes dij −2 = dij 2 + εi 2 = dij 2 + Ri 2 2 (3) where Ri is the estimated

radius of the area represented by a demand point i. The larger the radius the greater the distance the customer has to travel to the facility, with additional costs to him/her. Hence, based on the latter, authors show that the equation for Uij can be corrected as follows $Uij = Ajdij -2 = Ajdij$ 2 + Ri 2 2 (4). While no argument is made against the distance correction enhancement presented, it is evident nonetheless that saturation, as important as it is, is missing altogether in their model formulation and analysis, with only a very marginal use of it. Thus in an effort to improve on the model the paper introduces the increasing function of Saturation S (sij) affecting the customer's marginal utility and the facility facing the spatial saturation where sij is the level of saturation in its various forms faced by customer i and the facility (store/mall) at j in the area for j = 1, . . . n. Hence $Uij = F(Aj)\ S\ (sij)H(dij)$ (1*). Now building on the new approach, it is proposed that expression (2) above can now be written as $Uij = Aj\alpha Sij$ $\Psi dij\beta$ (2*) where the parameter Ψ to which ij s is raised is equal to the IRS value of the particular urban area in question. Thus the larger the value of Ψ is, the more detriment that exists for existing vendors as well as for new entrantsthen let ij p be the probability frequency of customer situated at i using the facility at j. This frequency, they claim, equals the relative utility of facility at j compared to other facilities available in the network.

4.1 The Condition can be expressed as Following

= (5) where E is the set of existing competitive locations and J is the set of available unoccupied nodes. Therefore, represents the share of market i captured by the facility at j. Now with the new approach proposed, one has that

$$p_{ij} = \frac{U_{ij}}{\sum_{k\in E}^{n} U_{ik} + \sum_{k\in J}^{n} U_{ik} - \sum_{k\in S_{ij}}^{n} U_{ik}} \quad (5^*)$$

where is the saturation level associated with each of the competitive location existing in the area. Hence expression $\sum_{k\in s_{ij}}^{n} U_{ik}$ represents the sum of the negative utilities (in the eyes of the customer and the retailer looking to locate in the area) associated with these competitive locations. Thus the original expression (5) above has been replaced by (5*) to account for retail saturation. Finally, the paper will not delve into the mathematical programming formulation nor will it refer to their use of max-min ant system solution as it is not part of the scope of this paper. Therefore, average distance to the store location is but one of several cost factors, but in the big city, the choices may vary considerably and so does customer utility. A store that might seem a much better option for consumers if the Euclidean distance were the measure of choice (much more attractive in terms of distance between customers and retail facility) following the traditional criteria expressed, might not be so. This because in the real world of the big city, once the other costs are taken into account, it might seem much more reasonable and convenient to drive farther to another store or mall to avoid grueling traffic jams, toll costs, parking saturation or the lack of parking altogether. Saturation, as already seen, can directly impact the attractiveness of retail facilities and market share captured by competing retail facilities. Moreover, retail saturation must be weighed into the analysis as greatly affecting the store attractiveness and can even override the distance factor when other factors are considered. If on the other hand one were talking about a small town somewhere that has a few stores and open streets with very limited traffic and no freeway, traffic jams and tolls or parking tickets to worry about, then life would be ideal and if the few stores in town were similar in terms of supply and price range, then distance alone would surely be the factor to decide on, but ideal or simplistic scenarios are not what consumers face in the modern city. Assuming a Utility function U for consumers where U = f (x1, x2,...,xn) where factors x1, x2…, up to some nth x all contribute to a consumer's utility. As discussed, saturation is one of these factors, but as this paper contends, it weighs more than others when analyzed from every angle. When talking about maximizing the consumer's utility function, one is saying that consumers are trying to attain their goals when faced with choices and the costs of these choices or options, as each of these choices implies costs and benefits and the rational consumer will make the best decision possible given the circumstances he/she is in. It is common knowledge that the market share captured by a retail facility is directly related to its attractiveness, as and many other researchers have repeatedly pointed out. Notwithstanding the latter, one contends that saturation impacts directly both the consumer's utility function and the retail facility's attractiveness and therefore its patronage; hence its utility is also negatively affected. For this reason and using the basic

model provided but incorporating changes to account for retail saturation as a variable, which impacts facility attractiveness and its market share, a different approach to model the problem is presented next. Following notation employed where n is the number of communities (each community represents a small urban area) Xj is the attractiveness of competing retail facility j (for j = 1, . . . ,k) and Sij is the level of saturation faced by customer i and facility (store/mall) j for j = 1, . . . ,k. Saturation may then be represented simply by its classical saturation index IRS = C∗RE where RF = Ψ or one can look at the function of Saturation S (sij) introduced earlier where sij is the level of saturation in its various forms faced by customer i and facility (store/mall) j in the area for j = 1, . . . ,n and so one may treat saturation as a more complex factor, where several sub factors intervene (various forms of saturation). If this option is chosen, one may see S as a sum of subfactors affecting customers and retail facilities, some of which were already discussed earlier when pointing at the multifaceted nature of Saturation. Offering an additional insight into the saturation issue from the customer's perspective**,** one can see that, unlike the gravity based models vastly used in traditional mainstream analysis, a better and more realistic assessment can be made by considering an insightful approach already discussed herein; this is that a relationship exists between the following factors: Market share captured by retail facilities ; the level of saturation associated with facility (store/mall) j which concerns customer I; the number of competing retail facilities in the urban area k; customers buying power , the metropolitan distance (with all its complexities) between customers and retail facilities ; and retail facility attractiveness . Hence the facility attractiveness and thus the market share captured by facilities in total dollar sales is now a function of both, saturation and distance and respectively meaning that both factors equally relevant intervene in the customer's choice. Moreover, both ought to be used and saturation is a factor not only associated with facility j as retail industry analysts and researchers always present but with customer i at any given time as well, just like distance is, realizing that the interrelationship between i and j is ultimately what shapes saturation . Please note that, the attractiveness of competing retail facility j (for j = 1, . . . , k), is treated as a factor of both saturation and distance in this new approach, in an effort to draw a different angle and shed more light onto the problem. This based on the fact that with information technology permeating and reshaping every aspect of people's lives, in today's market it is reasonable to think that in a given area of town or urban community, market information is readily available and each competing retailer knows enough about everyone else in the market as do consumers; henceforth one may safely assume that there is no "unknown attractiveness" in new and existing facilities in the community. The functional relationship among the above factors is shown as follows. The probability p_{ij} that a particular customer coming from community I patronize retail facility j is then

$$p_{ij} = \frac{\frac{X_j}{S(s_{ij})S(d_{ij})}}{\sum_{j=1}^{k} \frac{X_j}{S(s_{ij})S(d_{ij})}} \tag{7}$$

where $\sum_{j=1}^{k} p_{ij} = 1$ in ideal conditions (with no saturation or very low levels of it and reasonable distance conditions). However, based on earlier discussions, this is hardly the case most of the times in large urban areas and suburbs, plagued with all sorts of congestion, logistical barriers and an array of saturation conditions. Realizing this one may rethink the assumption above that the probability that a customer from community i patronizes retail facility j is 1 and instead use a more realistic approach whereby can be estimated/evaluated empirically and whose value range should vary anywhere between 0 and 1, depending on factors and) affecting both retailers and customers but in a different manner. If this were done none of the simplification done in their approach could be possible. Moreover, one would expect a much more complex scenario to be dealt with. However, this will be left to others to pursue, limiting ourselves to show that another view on the matter is indeed possible and worthy of study. Thus for the sake of simplification and to illustrate the model following the authors' con

vention, allowing the simplification $\sum_{j=1}^{k} p_{ij} = 1$ in the approach proposed to hold as well albeit insisting that a more realistic route should be explored. Now according to the authors, the market share attracted by retail facility

j; is the proportion of market share (total sales) captured by retail facility j, for all j = 1, . . . ,k where

$$m_j = \frac{\sum_{i=1}^{n} B_i P_{ij}}{\sum_{j=1}^{k}\sum_{i=1}^{n} B_i P_{ij}} \quad \forall j = 1\ldots., k$$

On their model, have

$$\sum_{j=1}^{k}\sum_{i=1}^{n} B_i P_{ij} = \sum_{i=1}^{n} B_i \sum_{j=1}^{k} p_{ij} = \sum_{i=1}^{n} B_i$$

assuming of course that $\sum_{j=1}^{k} p_{ij} = 1$ then becomes

$$m_j = \frac{\sum_{i=1}^{n} B_i p_{ij}}{\sum_{i=1}^{n} B_i} = \sum_{i=1}^{n} b_i p_{ij} \quad \forall j = 1\ldots., k \qquad (9)$$

Thus, substituting equation (7)

back into Equation 9 leads to the expression below with being the saturation function.

$$m_j = \sum_{i=1}^{n} b_i \frac{\frac{X_j}{S(s_{ij})S(d_{ij})}}{\sum_{j=1}^{k} \frac{X_j}{S(s_{ij})S(d_{ij})}} \quad \forall j = 1\ldots., k \qquad (10)$$

The k facilities' attractiveness represented by X_j, for j = 1, . . . , k are now to be determined based on distance and saturation, not just distance alone, as shown in the equations above. Equation 10 is therefore a simultaneous set of k non-linear equations with k unknowns, as shown. The rest of the formulation follows the same logical path offered by the authors. This set of equations has the following two properties: Homogeneity, whereby if the X_j's are multiplied by a common non-zero factor, so that the respective market shares do not change. This is because both the numerators and denominators in equation (10) are multiplied by the same factor.

5. Conclusions and Final Remarks

The retail industry is extremely competitive and facility location plays an important role in the competitive strategy of retailers. Until now, the mainstream literature on competitive facility location analysis has kept the discussion centered on facility attractiveness based on distance and also considering square meters of floor space as determinants of market share and profitability. However, it has failed to explore more closely and in-depth such factors as saturation in its various facets and how it acts as hindrance to firms wanting to locate into a market, affecting customer shopping frequency based on overcrowding and other detrimental effects derived from saturation itself. Saturation is one important variable that begs to be analyzed further in its various dimensions and so far it has been overlooked by mainstream analysis. Furthermore, saturation is a factor affecting the marginal productivity and revenue stream of retailers and the marginal utility of consumers when analyzed systemically, in the context of its complex interrelationship with other factors. Thus the approach of this paper has been threefold. First to discuss saturation in general terms and in retailing in particular, noting how absent it has been from the competitive facility location discussion thus far are too limited in scope and fail to bring saturation into the core of the mathematical analysis altogether, although they help bring some insight into the matter and are a timid step forward in the right direction. Secondly the new approach presented here discusses saturation in its different forms, by reviewing the concept of saturation applied to the retail industry and make a parallel with competitive facility location models presented by mainstream authors, such as Ghosh and Craig (1983) and their models are examined for their suitability to introduce saturation in the mathematical modelling arena and show how these may change if saturation is duly introduced in the mathematical discussion, in an attempt to improve on their approaches. Researchers working on competitive location decision making should focus their attention on saturation as a complex, multifaceted variable that affects new entrants and incumbents as well as consumers, all in a different manner and degree, depending among other things on spatial conditions. Saturation, as it has been shown, may be looked upon as a threshold point beyond which decreasing marginal utility to consumers and diminishing business activity affecting both customers and retailers occur. Profitability will also

suffer at the hands of saturation with diminishing returns albeit not in an immediately visible way but in a rather silent, disguised and pernicious mode, as a direct result of the detrimental conditions imposed to consumers and retailers by the saturation function. Unlike the approaches by other authors shown here, where saturation is only a number like IRS, the new approach presented proposes to go further and to explore saturation in the form of a variable and also a function involving several variables that comprise the multifaceted role of saturation, each one of them adding its own form and quota of saturation to the competitive location problem. Thus saturation is hardly a one-sided problem but a two-fold one, which affects both retailers and consumers alike although in different ways and degrees. This is essential to understand in order to further research into the subject by adopting a different optic, contributed here, in an effort to advance such an important and complex concept as retail saturation and its role in competitive location theory.

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