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More Than Words: An Integrated Framework for Exploring Gestures' Role in Bilinguals' Use of Two Languages for Making Mathematical Meaning

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Abstract Gestures play a role in perception, production, and comprehension of language and have been shown to differ cross-linguistically and cross-culturally in aspects of performance and form-meaning relationships. Furthermore, gestures can serve as analytical tools to access tacit embodied-imagistic mathematical meanings that add to verbal-linguistic dimensions of meaning. At the same time, language plays important roles in interaction and cognition, influencing bilinguals' learning of mathematics. Still, there is only very little research attending to the use of gestures of multilinguals as means to better understand the relationships between their language use and their mathematical thinking. This paper builds on research on multilingualism and on gestures—related and unrelated to mathematics education—to motivate and develop a framework for understanding better mathematics thinking and learning of multilinguals through integrating gesture analysis as related to languages, culture, and the use of registers. The application of this framework will be illustrated through two case studies in which we analyse interview data of a bilingual student and a bilingual mathematics teacher—focusing on gestures and language use while talking about the mathematical concept of 'power'—or exponents—in Farsi (Persian) and in English. From analyzing the gestures' form-meaning relations and their functions as related to hybrid language practices, we hypothesize on the vernacular and mathematical context as activated in both speech and gesture and on how it relates to mathematical meaning. From this, we draw practical implications for multilingual mathematical learning contexts and discuss implications for research on multilinguals' mathematical thinking and learning.

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Résumé Les gestes jouent un rôle dans la perception, la production et la compréhension du langage et il a été démontré qu'ils diffèrent d'une langue et d'une culture à l'autre en ce qui a trait aux aspects de la performance et les relations entre la forme et le sens. De plus, les gestes peuvent servir d'outils analytiques pour en arriver à des significations mathématiques tacites, représentatives et imagées, qui s'ajoutent aux dimensions verbales et linguistiques associées au sens. Parallèlement, le langage joue un rôle important dans l'interaction et la cognition, influençant l'apprentissage des mathématiques par les personnes bilingues. Pourtant, il n'existe que très peu de recherches portant sur l'utilisation des gestes dans le but de mieux comprendre les relations entre l'utilisation du langage et la pensée mathématique chez les personnes plurilingues. Dans cet article, on cherche à approfondir la recherche existante sur le plurilinguisme et la gestuelle, qu'elle soit liée ou non à l'enseignement des mathématiques, afin de motiver et d'élaborer un cadre permettant de mieux comprendre la pensée et l'apprentissage des mathématiques des personnes plurilingues par l'intégration d'une analyse des gestes en rapport avec les langues, la culture et l'usage des registres. On illustre la mise en application de ce cadre en utilisant deux études de cas dans lesquelles nous analysons les données recueillies dans le cadre d'entrevues effectuées avec un étudiant et un professeur de mathématiques bilingues, en nous concentrant sur les gestes et l'utilisation de la langue lors d'une discussion portant sur le concept mathématique de « puissance» ou d'exposants, en farsi (en persan) et en anglais. Partant de l'analyse des relations entre la forme et le sens des gestes et leurs fonctions à l'égard des pratiques langagières hybrides, nous émettons certaines hypothèses sur le contexte vernaculaire et mathématique, tel qu'il est mis à contribution à la fois dans le discours et la gestuelle, et sur la manière dont il est lié au sens mathématique. Nous formulons des implications pratiques pour les contextes d'apprentissage des mathématiques qui sont plurilingues puis nous commentons les incidences pour la recherche sur la pensée et l'apprentissage des mathématiques par les personnes plurilingues.

Keywords Gestures · Multilingual conceptualisations · Hybrid language practices · Meaning making · Exponents

Introduction

Ever since Pimm's work on "speaking mathematically" in the 1980s, the role of language in mathematical thinking and learning has increasingly gained attention in mathematics education (Pimm, 1987; Morgan et al., 2014). Considering its potential communicative and cognitive functions (Maier & Schweiger, 1999), language affects processes of learning mathematics on social and individual levels through its contribution to shaping the social interaction between teacher and the students and among students, but also influencing the students' mathematical thinking, subsequently feeding back into interaction.

Mathematics is taught and learned in two or more different languages in many parts of the world (see Barwell, 2009), examples including the USA (e.g., Moschkovich, 2007), South Africa (e.g., Setati & Adler, 2000), Wales (e.g., Jones & Martin-Jones, 2004), Malta (e.g., Farrugia, 2007), Spain, (e.g., Gorgorió & Planas, 2001), Australia (e.g., Ellerton & Clements, 1991), Canada (e.g., Barwell, 2014, 2015), Papua New Guinea (e.g., Clarkson & Galbraith, 1992), Iran (e.g., Parvanehnezhad & Clarkson, 2008), and the UK (e.g., Farsani, 2016). There is substantial evidence that the language of 'doing mathematics in the classroom' can vary cross-culturally (Gorgorió & Planas, 2001; Farsani, 2016; Morris 2021). Research in multilingual education sites not only raises awareness to the linguistically and culturally diverse mathematics classrooms (Barton, 2008; Parvanehnezhad & Clarkson, 2008; Farsani, 2015a) but also explores how different languages and cultures provide particular epistemological access offering different views on aspects of mathematics (Barton, 2008; Farsani, 2015b; Moschkovich, 2007; Morris, 2021; Krause, 2019 and Krause & Wille, 2021 for sign languages). The relevance of understanding better the communicative

and cognitive functions of language becomes hence amplified in multilingual learning settings and with respect to multilingual learners making use of their available resources for meaning making in mathematics teaching and learning (Barwell et al., 2007).

Research in psycholinguistics indicates that the languages we speak shape our thinking and influence how we see and understand the world on a structural and conceptual level (Boroditsky, 2011; Lucy, 1996)—a non-deterministic version of the 'linguistic relativity hypothesis', also known as the Sapir-Whorf hypothesis. This idea has been taken up in mathematics education in the past, suggesting how different languages structure and express mathematical concepts differently and might thereby emphasize and favour a certain approach to perceiving and understanding those concepts (e.g., Austin & Howson, 1979; Miura et al., 1994; Poisard et al., 2015; Krause, 2019 and Krause & Wille, 2021 for sign languages). For example, different languages provide different structures in naming multi-digit numbers, supporting or hindering early understanding of and dealing with these numbers (e.g., Miura et al., 1994; Ngan Ng & Rao, 2010). Prediger and colleagues (2019) built on a general structural distinction of conceptual organisation within languages suggested by Leung (2017) and applied this to an analysis of conceptual understanding of fractions in Turkish-German bilinguals in episodes of code-switching. Both these languages show structural differences in referring to fractions in that they favour either a synthetic approach of thinking from the parts to the whole, reading fractions as numerator-denominator (German), or an analytic one thinking from the whole to the parts, naming first the parts the whole is divided in (Turkish). Their findings indicate that some bilingual learners can activate synthetic and analytic nuances of the part-whole concept as coexisting while using both languages side by side, becoming expressed across both languages in what they call a 'bilingual connection mode'. Their study suggests important insights about bilingual learners' conceptual organisation and their linguistic and cognitive resources available in mathematical talk.

In this paper, we claim that the analysis of bilingual learners' gestures might provide an additional perspective on this matter. Considering the deeply intertwined structural and semantic relationships between gesture and speech production, understanding the role of language in bilinguals' mathematical thinking and learning needs to take into account more than what is said. Hand gestures—understood as spontaneous movements of hands and arms while speaking (McNeill, 1992), without fulfilling some practical aim or action—have been integrated widely in the past to consider the multimodal nature of interaction and communication in mathematics learning (e.g., Arzarello et al., 2009; Edwards et al., 2014; Krause, 2016; Rasmussen et al., 2004), in particular gestures' intertwined relationship with speech production and thinking (see e.g., Kita et al., 2017; Salle & Krause, 2021). Forming an integrative system, "gesture, together with language, helps constitute thought" (McNeill, 1992, p. 245), with gesture analysis used in psycholinguistics to refine versions of linguistic relativity (McNeill & Duncan, 2000). This understands the linguistic repertoire as encompassing speech and gesture as they concern "different sides of a single underlying mental process." (McNeill, 1992, p. 1) McNeill and Duncan emphasize the bimodal character of language as an opportunity to get a more comprehensive understanding of a gesture-speech utterance: "Each modality, because of its unique semiotic properties, can go beyond the meaning possibilities of the other, and this is the foundation of our use of gesture as an enhanced window into mental processes." (McNeill & Duncan, 2000, p. 144) Gestures can thus serve a double role in understanding better processes of thinking and learning mathematics: On the one hand, they can serve as an analytic tool for the researcher in that they provide embodied-imagistic access to the mathematical meanings activated by the speaker and represented in their utterance and thereby, to nuances of tacit conceptualizations activated in context (Edwards,2009). On the other hand, they can become themselves part of the multimodal interaction and contribute to meaning making as a semiotic resource (e.g., Arzarello et al., 2009). While researchers interested in the bilingual mathematics classroom already took notice of the important roles gestures can play in social learning processes, focusing on communication and discourse (e.g., Castellón & Enyedy, 2006;

Ng, 2016; Shein, 2012; see Robutti et al., 2022, for an overview), they largely leave out the individual dimension of the bilingual whose mathematical thinking and conceptual understanding are influenced by their languages and cultural background. Considering that bilinguals' use of different languages might afford both conceptual understanding and its expression differently, the close relationship between language and thinking, and gestures' potential as "window into mental processes" (McNeill & Duncan, 2000, p. 144), we propose a more systematic integration of the analytic potential of gestures to better understand the language-related complexities of bilinguals' sources of meaning (Barwell, 2018) and resources for meaning making.

This paper thus focuses on gestures as means to access bilinguals' mathematical thinking as related to their use and coordination of different languages. Acknowledging the role of language and culture for mathematical understanding as well as for gesture production and communication, we will present a framework for analyzing bilinguals' gestures in mathematical talk that takes into account the bilinguals' unique backgrounds for developing and expressing mathematical meaning. We will then illustrate the application of this framework in two case analyses of two bilinguals'—a student and a teacher—gestures while talking about the concept of 'power' during interviews. In these analyses, we will see how the speakers' gestures differ in their referential links depending on their use as being co-expressive to them using Farsi or English. As this goes beyond structural differences across languages, gesture analysis adds to an understanding of cultural and linguistic embedding of mathematical terminology in languages used in multilingual settings.

Language as More Than Words: First Theoretical Considerations Towards an Integrative Framework

This section will provide the foundation of our framework for integrating gesture analysis in bilinguals' mathematical talk by understanding gestures as related to the linguistic, cultural, and mathematical aspects of meaning making.

Multilinguals' Repertoires and Sources for Meaning Making

Bi- or multilingual learning settings are often observed with an interest in how students shift between two or more linguistic systems within single communicative events—a practice known as code-switching (Gumperz & Hymes, 1972). Furthermore, attention is drawn also to what someone *does* with their linguistic repertoires, that is, how languages can be perceived as fluid resources in meaning making practices rather than just as codes with solid boundaries, a perspective that developed into the ideas of 'polylingualism' (Jørgensen, 2008), 'metrolingualism' (Otsuji & Pennycook, 2011), and 'translanguaging' (Blackledge & Creese, 2010; García & Wei, 2014).

Adopting the broader notion of 'hybrid language practices' (Gutiérrez et al., 1999; Moschkovich, 2019) takes into account the language practices a bilingual engages in when activating the bilingual mode (Grosjean, 2001, 2010), that is, using both languages side by side. This perspective understands bilingual learners as unique language users who draw on a wide linguistic repertoire that is not confined by static language boundaries but instead considers holistic bilingualism in which language use reflects underlying, distinguishing, and nuanced experiences with the respective language (MacSwan, 2017). Switching between languages is then an expression of a wider and more holistic coordination of these experiences and the related resources available to the speaker.

Considering the dynamic and reciprocal relationship between language use and thinking, Prediger et al. (2019) expanded on Grosjean's (2001) idea of bilingual modes by what they call *bilingual complementary mode* and *bilingual connection mode:* The former "could appear when learners combine

language-related conceptualisations, each in their own language" (Prediger et al., 2019, p. 192), the latter when "individuals refer to language-related nuances of conceptualisations in all languages and when they combine aspects of different nuances" (p. 192). Prediger et al. claim that conceptual understanding benefits from the connection mode in that learners become able to activate the whole range of their conceptual understanding across the whole range of their linguistic repertoire and can thus more efficiently draw on the linguistic and cognitive (re)sources for meaning making at their disposal (see also Schüler-Meyer et al., 2019).

As that, such research on the interplay between language and mathematical thinking contributes to better understand the dimensions of language as a resource in its epistemic value (Planas, 2019) in that understanding multilinguals' repertoires as resources for meaning making includes their potential as source of meaning (Barwell, 2018) as well as the ways in which this potential develops across languages (Planas, 2018).

The Interplay Between Culture, Language, and Registers for Mathematical Meaning Making

The relation between mathematics and languages is inextricably interwoven through the linguistic notion of registers (Halliday, 1975, 1978)—that is, a particular use of language in a specific situational context. With respect to mathematics, this concerns "the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (Halliday, 1978, p. 195).

Words used in the English mathematics register can have a particular meaning in the context of mathematics only, but can also have a meaning in both mathematics and social vernacular contexts. For example, the terms 'hypotenuse' is an example of the English mathematics register that has a specific place and value only within the context of mathematics. On the other hand, terms like 'ring', 'field', or 'mean' are used both in mathematics and in vernacular contexts and the meaning of any of these words can change depending on the context. In their study with mathematics teachers in Papua New Guinea, Edmonds-Wathen et al., (2019) stress the role of cultural identity as it might influence mathematical activity carried out in vernacular language—in their case Tok Ples—being shaped by and within the culture. Switching between vernacular terms across two or more languages in mathematical talk hence also means coordinating multiple vernacular contexts linked to multiple cultural identities, as these affect facets of mathematical understanding.

One of Halliday's main claims in relation to mathematics registers concerns how talking about and doing mathematics in different languages influences mathematical understanding differently, as is pointed out by Barwell (2008) in the following:

> Different languages, through their semantic structure, stress some meanings more than others. The mathematics registers of different languages, therefore, stress different mathematical meanings. This may mean that students of mathematics in different languages develop differing awareness of a given aspect of mathematics (p. 318).

Considering this, mastering the mathematics register (in a particular language) does not only help interlocutors to describe and express mathematical properties and relationships specifically and to express mathematical meaning in that particular language (Pimm, 1987)—moving between mathematics registers in several languages might provide different perspectives on mathematical ideas that eventually need to be coordinated in a bilingual connection mode in order to engage with mathematics across languages successfully (see Prediger et al., (2019) and their example of nuances of fraction understanding advantaged through language structures in the German and the Turkish mathematics register).

Considering that dealing with mathematics goes beyond verbal mathematics registers but includes various semiotic sources of constructing mathematical meaning, Prediger and Wessel (2013) consider a wider

social semiotic approach on registers and the interactions between them in their model for relating different registers and representations (p. 438). For this, they marry Halliday's linguistic notion of registers and Duval's notion of semiotic registers (Duval, 2006) to include not only verbal aspects of referring to mathematical ideas across languages, but also the coordination of their different representations as crucial for combining different perspectives in meaning making (e.g., Bruner, 1966; Duval, 2006). Gestures have been considered in this model in the sense of concrete representations (Prediger et al., 2016), without considering the implicit and tacit ways in which they can reflect a speaker/gesturer's meaning—both in relation to the verbal register and on its own merit—then becoming a potential source of meaning making in mathematical communication and interaction.

While the first part of this section made a case for considering multilinguals' mathematical meaning making within the deeply intertwined relationships between language, linguistic registers, and culture, linking this to semiotic registers as considered in the model by Prediger and Wessel (2013) allows for framing gesture analysis as related to bilinguals' mathematical meaning both against the background and also in interaction with the languages and registers at use.

The following sections will shift perspectives to elaborate on previous research on relationships between mathematical thinking and gestures as well as framing gesture analysis.

Embodied Mathematics: Bimodal Minds, Bimodal Communication

Mathematics educators are engaging with gestures as embodied resource in mathematics both as a means to access mathematical thought and considering their roles in mathematical thinking and learning (e.g., Alibali et al., 2014; Edwards, 2009; Hall & Nemirovsky, 2012; Krause & Salle, 2019). Within the last few decades, mathematics education increasingly considers mathematical thinking and learning as embodied processes, in which mathematical understanding is shaped by the experiences we make in the physical and cultural world (e.g., Lakoff & Núñez, 2000; Nemirovsky, 2003; Varela et al., 1991). At the same time, our bodily expression reflects these aspects of our understanding and thinking through both verbal and non-verbal ways—through language and through gestures (e.g., Edwards et al., 2014; Goldin-Meadow, 2003; Hostetter & Alibali, 2018). In particular, both can reflect how mathematical meaning can be built on concrete experiences in metaphorical ways, influenced by our cultural and linguistic backgrounds (Lakoff & Núñez, 2000; Edwards, 2009).

Acknowledging this, mathematics education scholars adopted gesture analysis as a tool to understand better mathematical thinking in various contexts, building on previous work in fields like psychology and cognitive linguistics (e.g., Kendon, 1980; McNeill, 1992; Goldin-Meadow, 2003). Edwards (2009) used the model of conceptual blending on a gesture corpus collected from interviews on fractions with prospective teachers as an analytical tool to understand better the gestures produced in mathematical talk and to capture the nature of their reference. While she found many of the gestures contributing to the mathematical explanation on fractions metaphorically, her results furthermore provide a framework to capture the sources of meaning making employed when talking about a mathematical idea, may they be based on physical interaction with manipulatives (iconic-physical reference), or referring to the semiotic engagement with written symbolic mathematics (iconic-symbolic reference). Goldin-Meadow et al. (1993) describe how gesture-speech mismatches—instances in which gesture and speech represent complementary or even contradictory information—can reveal a student's readiness to progress in learning. Zurina and Williams (2011) focus on the role gestures might have for the person gesturing in mathematical problem solving and relate this to how they might substitute inner speech, adding an imagistic dimension to meaning making. Healy and Fernandes (2011) engage with gestures beyond the purpose of communication in their work with blind students in processes of making mathematical meaning. While the blind students did not use the gestures to communicate with their peers, Healy and Fernandes argue that, "in term of the production of mathematical meanings, gestures serve as a means

to create embodied abstractions associated with the identification or definition of general properties and relationships experienced as concrete in a particular setting but applicable beyond." (p. 172).

Research like this shows the potential of gesture analysis to understand better how individuals conceptualise mathematical ideas, and how they employ gestures as a resource for individual meaning making and for elaborating their mathematical ideas and explanations.

However, essentially all research on gestures related to mathematical thinking has been carried out in monolingual contexts. Although the few examples carried out in bilingual settings unfortunately do not take into analytic account the specificities of gesture use as related to their use of two languages (e.g., Domínguez, 2005), they nonetheless contribute greatly in providing a rationale for investigating gesture use of bilingual learners. Communicative and cognitive functions of gestures in mathematical thinking and learning in bilingual contexts are however still an uncharted field of exploration.

Differences in Gesture Use Across Languages and Cultures

While there is generally remarkable little research on the peculiarities of gesture use of bilingual individuals (Gullberg, 2012), literature points out differences of gesture use across monolingual speakers of different languages and with different cultural backgrounds, for example, concerning frequency of gesturing, typical size of gestures, the meaning of emblematic gestures, and the employment of the gesture space (see Kita, 2009 and Gullberg, 2012 for an extensive review of the literature).

Gesture studies typically distinguish form-meaning relationships of gestures in degrees of conventionalization and situatedness in co-occurring speech (McNeill, 1992, p. 37; see Fig. 1).

While gesticulations need the context of speech for interpreting their reference, the meaning of emblems is culturally integrated and conventionalized and often differs cross-culturally (see Kita, 2009).

Aside from emblems showing differences in gesture use across cultures and languages, Kita (2009) furthermore describes differences in gesture use grounded in cross-cultural differences in perception and conceptualisation. Notably, he also points out variations based on linguistic diversity across cultures as they are linked to concept organisation:

> Different languages are spoken in different cultures, and languages have different lexical and syntactic resources for expressing thought. Even when describing exactly the same event, languages may differ as to what aspects of the event are encoded lexically or left out from the description or as to how the words describing various aspects of the event are distributed syntactically. (p. 154)

Combined with the claims of linguistic relativity that the language we speak shapes our thinking, it also shapes the ideas that we choose to express as contextually related to a given topic. Considering gesture analysis will therefore allow us to better understand bilinguals' mathematics expressions beyond words, following Gullberg (2012) in that "gesture […] reflect information considered relevant for expression (what to say) as well as its linguistic encoding (how to say it), with cross-linguistic consequences." (p. 420).

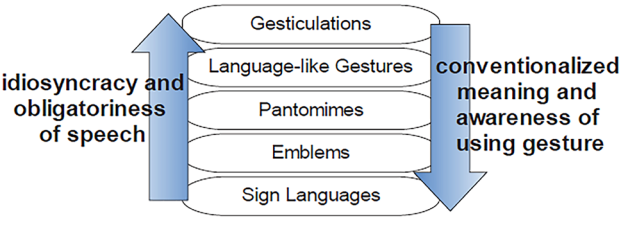


Fig. 1 Representation of Kendon's continuum (Krause, 2016, p. 27)

Implications for the Analysis of Gestures in Bilingual Cross-cultural Mathematical Talk

An often-adopted typology for describing the form-meaning relationships of co-speech gestures distinguishes representational gestures in their iconic, metaphoric, and deictic dimensions (McNeill, 1992). The *iconic dimension* concerns how gestures bear some resemblance to an aspect of a concrete referent, *metaphoric gestures* are like iconic gestures but refer to an abstract idea in a figurative sense (such as 'presenting an idea', by moving forward the open hand, palm directed upwards, when talking to somebody), and *deictic gestures* refer to something concrete by directly pointing at it with a finger or the whole hand, or refer to something in an abstract way if it is not directly present (such as pointing to an empty seat in the classroom when asking about a student who is usually sitting there). As mentioned in a prior section, Edwards proposes a distinction of iconic gestures in the context of mathematics in *iconic-physical* and *iconic-symbolic*, as they embed the source of the gestures differently in mathematical activity with manipulatives or with conventionalized notations (Edwards, 2009). Gestures in mathematical talk then align in meaning with the co-expressive speech in that mathematical ideas become linked to action, written symbolic or graphic representations, or metaphors that represent aspects of the mathematical content in a mediated, abstract way.

Considering the cross-linguistics and cross-cultural differences in gesture use in form and meaning, the interpretation of gestures in bilingual contexts in mathematical talk needs to take into account gestures' relationship with the co-occurring speech with the cultural, linguistic, and domain-specific background information such as cultural identity and vernacular meaning of the terminology related to each language: The four general referential dimensions for representational gestures in mathematical talk hence need to be considered against the backgrounds of potential culturally influenced gesture use differing across languages (e.g., emblems), and language-related registers and conceptualisations. Figure 2 summarizes this background for gesture analysis specific to bilinguals' sources of meaning, the components distinguished here analytically while playing together in interpreting gestures' meaning and functions. The dotted lines express the interrelated nature of these different components in bilinguals' mathematical meaning making.

We will adopt this approach for exploring the role gestures might play in hybrid language practices in mathematical talk in the following two case studies.

Framework in Action: Two Case Studies of a Student's and a Teacher's Use of Bilingual Bimodal Mathematical Talk About 'Power' and 'Tavan'

The data presented here is part of a data corpus collected in the context of an unrelated study on multimodality and mathematical meaning making in multilingual classrooms in a bilingual school in the UK (Farsani, 2015a). This bilingual school is a type of community educational institution that functions outside of mainstream school hours, typically referred to as 'complementary schools' in the UK (e.g., Creese et al., 2008). These schools provide a space for performance of alternative languages, histories, and pedagogies (Farsani, 2016) and are often attended in order to learn the language that is associated with their heritage group. The learners can vary across their linguistic proficiencies; they can be first (second or even third) generations of immigrants or they can have a parent who is not from that particular heritage background. This particular British-Iranian complementary school offered learners the opportunity to learn mathematics in both *English* and *Farsi*. It is very common to observe that in these types of schools, the classroom teacher actively uses two languages by drawing upon their linguistic repertoires (Arthur, 1996), to motivate and engage with students (Blackledge & Creese, 2010).

To illustrate the application of the framework, two excerpts from interviews conducted by the second author with teachers and students of the school were chosen. Originally carried out to obtain a better

understanding of their general cultural, linguistic, and mathematical backgrounds, we consider this setting particularly suitable for our focus on the individual dimension of meaning making understood from looking beyond the bilinguals' spoken language. The cases show a bilingual student's (Ali) and a bilingual teacher's (Ebi) use of gestures when talking about the mathematical concept 'power (function)' in Farsi and English. In particular, we chose these examples to specifically address the interplay between language, culture, and registers as influencing the gesture analysis in mathematical talk across languages.

The topic of 'power functions'/exponents arose spontaneously in Ali's interview when asked about topics in mathematics he likes and dislikes. The topics mentioned by the students have then been used as a baseline for the interviews carried out with the teachers, before any multimodal analysis of the student data has been conducted.

Power functions are defined as functions of the form $f(x) = x^a$ with the base variable element in $\mathbb{R}$ and the coefficient a a real number. In school contexts, a is mainly considered a positive integer, allowing an understanding of power functions as repeated multiplication of the base x with itself a times. The arithmetic operation of exponentiation hence leads to a rapid growth of the power function. Verbally, the term x^a is read as "x *(raised) to the power of* a" in English and "x *be tavan-e* a" in Farsi.

The cases of Ali and Ebi appeared suitable for an illustration as showing similarities in mathematical topic and gesture form-meaning relationships while at the same time giving opportunity for comparing differences in gestures' role in the hybrid language practices.

The analysis of these cases followed two streams: The first one concerned the interpretation of the form-meaning relationships of the gestures with respect to the co-occurring speech. For this, gestures have been identified and interpreted with respect to their relationship to 'what' concept is referred to in co-expressive speech and 'which' language is used to refer to the concept. This tentative form-meaning has then been linked to potential iconic and metaphoric meanings against the background of the components presented in Fig. 2, based on the language in use for and furthermore potential symbolic representations related to the mathematical concept. The second stream then concerned the integration of the use of gestures related to the use of two languages within the larger context of the gesturer's response

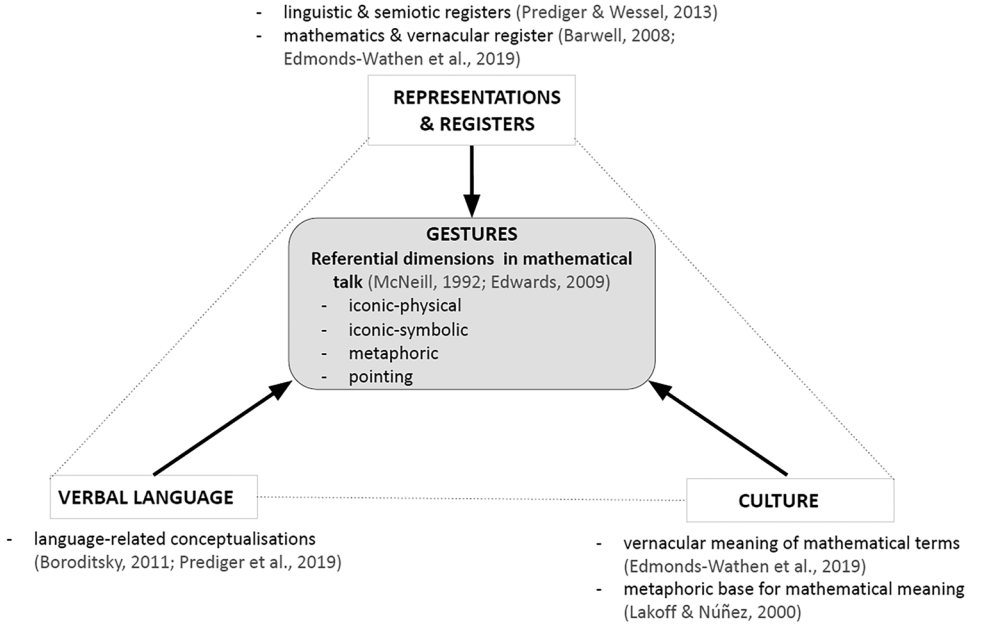


Fig. 2 Background for interpretation of form-meaning relationships of bilinguals' gestures in contexts of mathematical meaning making

to the interviewer's question. More concretely, we identify potential hybrid language practices and hypothesize about the functions gestures might play with respect to responding to the question (e.g., Kita et al., 2017; Krause, 2016; Salle & Krause, 2021; McNeill, 1992).

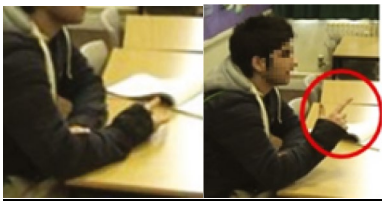
We will see that both Ali and Ebi employ different gestures across their languages in use, while both individuals seem to refer to similar underlying ideas in their gestures when using the same respective language. Furthermore, we consider potential functions these gestures might fulfill concerning the coordination of the two languages in hybrid language practices and how these functions differ between the two cases of Ali and Ebi.

In the transcripts, the use of Farsi and English is highlighted by the use of different colours. The gestures discussed are captured as images below each line with simultaneity of speech and gesture indicated by square brackets around the spoken part where the gesture is performed.

The Case of a Bilingual Student (Ali)

The first case presents an instance of using two languages in speech and gestures in an interview conducted with Ali, a native speaker of Farsi and a fluent L2 speaker of English (see Table 1). We will see how Ali exhibits two different gestures for the mathematical concept for power when switching between his languages—one for the Farsi '*tavan*', and another one for the English '*power*'.

Table 1 Bilingual Farsi-English transcript extracted from the interview with the student Ali and the English translation of the transcript

		Original Farsi-English dialogue	English version
1	I	What is it in particular about mathematics that fascinates you?	What is it in particular about mathematics that fascinates you?
2	Ali	Mainly, different subjects like finding the unknown in maths.	Mainly, different subjects like finding the unknown in maths.
3	I	mitooni ye mesaal baram bezani?	Can you give me an example?
4	Ali	masalan tavan, [tavan]-e riazi	for example power, [power] in mathematics
			
5		[Power]	[Power]
			
6	I	Tavan, oh okay	Power, oh okay
7	Ali	Tavan, equations	Power, equations
8	I	Oh okay, doroste	Oh, okay, right

Prompted by the interviewer, Ali provides 'power functions' as an example for a mathematical topic that fascinates him. He starts with a general mentioning of 'finding the unknown' (line 2) and seems to concretize this as the interviewer follows up with asking for an example (line 3). In the course of the next lines (4–5), Ali's example becomes increasingly specific in his spoken utterance, first offering more generally the vernacular '*tavan*' (4, '*power*'), then clarifying 'tavan-e riazi' (4, 'power in mathematics'), then using the English word 'power' (5), and eventually arriving at 'tavan, equations' (7, 'power equations') in a Farsi-English utterance, providing an example for a context to 'find the unknown in math' (2).

His gestures now tell us more than the words alone. In line 4, when Ali starts the elaboration of his example in Farsi, Ali touches his bicep as he synchronously utters '*tavan*' (Fig. 3). Within the specific Persian cultural context, this gesture of the upper arm emblematically refers to the notion of strength or being "strong" (Sparhawk, 1981, p. 445). Right after, moving to the English '*power*' (line 5), Ali uses a different gesture (Fig. 4), in which he performs a slight upwards movement that suggested a 'higher up' position as Ali displayed a hand gesture from the 'rest' (with his right hand being on the table; see Fig. 3 on the right) moved up towards his shoulder level. This can be interpreted as indicating the elevated position of the exponent in relation to the base in the written mathematical notation of 'something to the power of something' in an iconic-symbolic way (Edwards, 2009). Similar abstract pointing related to an exponent when squaring has been observed elsewhere before (e.g., Krause, 2016, p. 172).

There hence are two different gestures activated in two instances of referring to the same mathematical idea as Ali switches between the two languages, gesturing differently for '*tavan*' and '*power*' as he shifted to English. At the same time, Ali's '*tavan*'-gesture shows a different conceptual quality than the '*power*'-gesture in the context of mathematics in which it has been accomplished. Different from the latter, the former does not seem to refer to some conceptual aspect of power or exponents in the mathematics register, but rather seems to employ reference to vernacular meaning of '*tavan*' in Farsi. The vernacular meaning of the term '*tavan*' as it is used outside of the academic setting (and possibly where '*tavan*' was first encountered) has only one meaning, conveying the notion 'strength'. Interestingly, Ali accomplishes the '*tavan*'-gesture not in the more general first verbal use of '*tavan*', but in the second one, in which he explicates the reference to the mathematical context explicitly as 'tavan-e riazi' ('power in mathematics'), suggesting a link to the mathematics register and a potential language-related meaning associated with the mathematical concept.

However, the vernacular meaning of the English term '*power*' is rather ambiguous as it can be linked to various vernacular concepts, considering different uses in the contexts of, e.g., 'power cut', 'political power', and 'power and strength'. In Farsi, on the other hand, the meaning for '*tavan*'

Fig. 3 Ali's gesture for '*tavan*' ('in mathematics', line 4): Ali's left hand moves from its resting position on the table to his right biceps, with the right forearm resting on the table

Fig. 4 Ali's gesture for 'power' (line 5): His right forearm moves from its resting position on the table up with the index finger extended and pointing diagonally upwards

appears to be less ambiguous as it suggests only one vernacular concept, that is, the notion of 'power and strength', as it is reflected in the gesture also in the explicitly mathematical context. While the vernacular contexts of the term used for the mathematics context concern a non-tangible—one might say abstract—concept in both English and Farsi, the unambiguous vernacular meaning in Farsi might emphasize this association to 'strengths' as language-related nuance of conceptual understanding of 'power', reflected in the gesture even in the mathematical use 'tavan-e riazi' ('power in mathematics', line 4).

Furthermore, noticing the verbal Farsi-English combination of vernacular and mathematical meaning in 'tavan equations' right after his 'power'-gesture (line 5; Fig. 4), we speculate about a potential cognitive function of this gesture, in that it might *activate spatial-visual information* (Kita, et al. 2017; Salle & Krause, 2021)—a semiotic register that provides visual support for Ali to concretize his idea in his verbal expression. When being asked for an example for a mathematical context of finding an unknown in mathematics, Ali first only mentions the key word '*tavan*' before concretizing that he talks about 'tavan-e riazi' ('power in mathematics', line 4), nuancing his culturally/language-related understanding of this concept through the simultaneous gesture. He then switches to the English mathematical register of '*power*', with his gesture showing a nuance of an understanding that refers to a symbolic-notational aspect of the concept (line 5), before eventually specifying his example to 'tavan equations', the actual context in which 'finding and unknown' would become a mathematical activity, in line 7. With the final statement being formed in both languages, it seems that both parts of the idea—tavan and equation—have been formed in the two respective languages, but also that both the '*tavan*'-gesture and the '*power*'-gesture had their part in this formation as becomes verbally captured in the combination of 'tavan' and 'equation', mixing Farsi and English mathematics register. Considering that gestures might activate "spatio-motoric information for the purposes of speaking and thinking" (Kita et al., 2017, p. 246), Ali's power-gesture and its link to the symbolic notation might have helped him in accessing the idea of 'equations' as linked in this specific context, fulfilling a self-directed cognitive function (Salle & Krause, 2021).

The Case of a Bilingual Teacher (Ebi)

This second excerpt (Table 2) is taken from an interview conducted with Ebi, a native speaker of Farsi and a fluent L2 speaker of English. The analytical focus is again on Ebi's enactment of '*tavan*' and

Table 2 Bilingual Farsi-English transcript extracted from the interview with the teacher Ebi and the English translation of the transcript

		Original Farsi-English dialogue	English version
1	I	beonvane ye moalem, age shoma farad bekhay beri madrese va tavan dars bedi be bacheha, va ghablesh emshab mikhain ye moroori rooye iin chizhayi ke mikhain dars bedin bedin, ino be farsi hal mikonin ya be ingilisi hal mikoni?	As a teacher, if tomorrow you are going to teach the concept of powers and, let's say that tonight you are going to prepare for it, would you do the preparation and the teaching of the concept power in English or in Farsi?
2	Ebi	na, man e e power ro hamishe be ingilisi yad midam masalan	No, I eeerm always teach power in English. For example
3	Ebi	Two to the [power] of three. Fifteen to the power of something 	Two to the [power] of three. Fifteen to the power of something
4	I	Uhum	uhum
5	Ebi	man hamishe power-ro be ingilisi kar mikonam chon ke maa e e . . vaghti be bacheha yad midi bayad be ingilisi yad bedi va masalan vaghti migi	I personally work with a concept of power in English because erm erm students learn for example when you say
6	Ebi	[Power] of something 	[Power] of something
7	Ebi	ta mogheike bacheha moshkel nadashtan talash mikardam ingilisi beheshoon dars	As long as they understand the point and do not have a problem with, I try to use

Table 2 (continued)

		bedam bekhatere iinke	English as the medium of instruction because
8	Ebi	At the end of the day they go to a English school and they learn everything in English. So	At the end of the day they go to a English school and they learn everything in English. So
9	Ebi	e e vaghti ke moshkel, vaghti ke nemifahman motovaje nemishe, iin etefagh kei miofte, vaghti ke bacheha taze oomadan, e e bachehayi ke nemitoonan befahman ke chera power mishe tavan, vaghti inro motavaje nemishan, man be farsi inro beheshoon migam	but if there is a problem and the students do not understand what is going on due to the fact that they are new arrivals[1] or erm, it is only then when those students can't see the relation between for example 'power' and *'tavan'*, then I teach them in Farsi
10	I	Doroste	Aha, ok
11	Ebi	tozih midam	I'll explain it to them
12	I	Doroste	Aha, ok
13	Ebi	bad beheshoon migam ke manzooremoon az [tavan] chi eke 	Then I will tell them what do I mean by [*tavan*]
14	Ebi	Which is for example the same as two to the [power] of something. 	Which is for example the same as two to the [power] of something.

[1]Ebi probably means 'new arrivals in the UK' here

'*power*' in his mathematical talk. Similar to Ali's case discussed above, Ebi's gestures reveal how his understanding of the mathematical concept for '*power*' and '*tavan*' varies flexibly as he uses two languages.

After being asked by the interviewer which language he would choose to teach the concept of power ('tavan', as asked in Farsi) in the classroom (line 1), Ebi states his preference for teaching concepts in English (line 2) and justifies this with the everyday English learning context the students encounter at school (line 8). He goes on providing exceptions for when he switches to Farsi in his teaching: There are a number of students who are new arrivals to the UK and may not be proficient enough in English to being able to make appropriate connections between '*tavan*' and '*power*' in the mathematical context and therefore he "will explain it to them" (line 11), with his explanation being described more concretely by him in the following lines 13 and 14.

Similar to Ali, Ebi's gestures differ when referring to '*power*' and to '*tavan*'. Although the '*power*'-gestures in lines 3, 6, and 14 vary in their performance, they all share the common trajectory, feature of upwards movement of his right hand while verbally referring to the concept of '*power*' (e.g., Fig. 5), which can again be interpreted as a reference to the position of the exponent in relation to the base in an expression such as a^b. While saying '*tavan*', Ebi forcefully pushes his right closed fist into his slightly bent open left hand in front of his body in a downward movement, vocally stressed by a rise in intonation (line 13; Fig. 6). Although this gesture differs from Ali's '*tavan*'-gesture, it can also be interpreted in its culturally grounded emblematic reference to force or strengths (Sparhawk, 1981). The gesture is followed by an immediate code-switching into his following English utterance in which he mentions '*power*' in a mathematical context of 'taking something to the power of something' (line 14), together with the gestural reference to '*power*' similar to before, leaving his left hand in the same position as it is for the '*tavan*'-gesture. As Ebi moved flexibly between his languages, he exhibits differing gestural references to '*tavan*' (Fig. 6) and to '*power*' (Fig. 5), even within one ongoing description of his approach to explaining that both words refer to the same mathematical idea (lines 13–14).

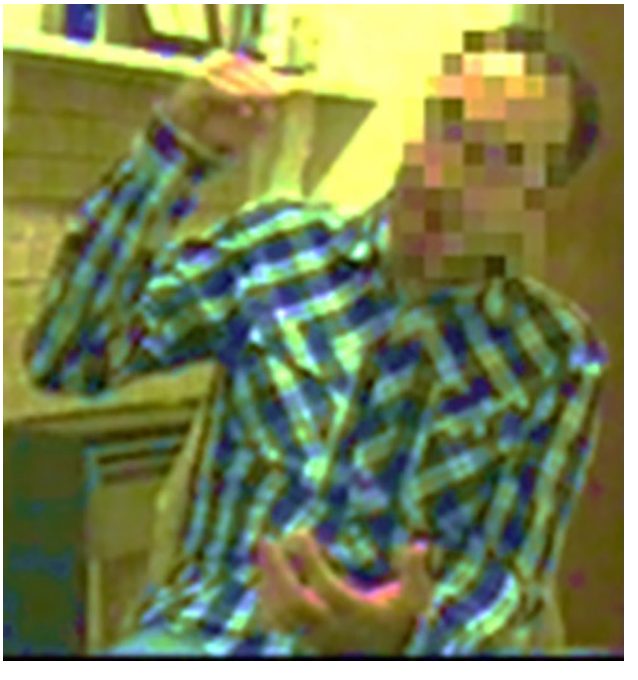


Fig. 5 Ebi's gestural representation for '*power*' (line 14): His right hand moves up from its final position in the gesture performed prior (line 13; Fig. 6) to eye height. In this movement, it does not only change location but also handshape, from a clenched fist to halfway extended

Fig. 6 Ebi's gestural representation for '*tavan*' (line 13): The right fist pushes down from chest height into his slightly open left hand extended with palm up in front of his body

Ebi seems to explicitly distinguish the English mathematics register as language of instruction from the everyday-use of language in vernacular contexts. However, he provides an example of how as a bilingual teacher, he can draw on his linguistic resources in a flexible manner (Blackledge & Creese, 2014; Farsani et al., 2022) to help his students coordinate the mathematics registers in both languages (see Krause & Farsani, 2021). Starting in the vernacular context of Farsi, he embeds his bimodal explanation in the students' cultural identity before switching to the English mathematics register in that he visually links the concept to the symbolic notation that might be known to the students already. Leaving his left hand in the same place for both gestures while switching language and the main components, he furthermore provides an embodied contextual link in this feature, encompassing not only both gestures and both languages, but also the vernacular context and the mathematical symbolic context referred to in the gesture-speech utterances. The way he switches from '*tavan*' to '*power*' in both modalities in his explanation might highlight that these arbitrary terms denote the same phenomenon in mathematics registers in two languages while establishing a link between the vernacular idea in Farsi and the conventionalized mathematics he wants to encourage them to express in English. His bimodal explanation can hence be considered to potentially support the bilingual understanding of the concept of power in a bimodal way.

Comparing Ali's and Ebi's Use of Gestures

Ebi's and Ali's gestural representations for '*power*' appear to have a similar iconic-symbolic form-meaning relationship: they share features of movement and positions of hands to each other, suggesting the position of the exponent in relation to the base in symbolic notation. In the bimodal English reference, they combine the semiotic algebraic register with the linguistic mathematical register. The gesture appeared to be grounded in the mathematical notation while at the same time stressing it, potentially fulfilling different functions in both cases.

Ali's and Ebi's gestural representations for '*tavan*' do not have the same form—as Ali touched his bicep and Ebi makes a clenched fist—but still seem to refer to the same idea of 'tavan' in its vernacular context in a metaphorical or emblematic way (Sparhawk, 1981). Here, gesture analysis needs to consider the cultural embedding and vernacular meaning of the term, as well as the speaker/gesturers' cultural identity.

An obvious but notable difference between Ali and Ebi concerns their statuses of student and teacher, respectively, also reflected in the potential functions of the gestures concerning the hybrid language practices in the two cases. As Ali is concretizing his idea while speaking, switching to English in speech and gesture might fulfill a self-directed cognitive function (Salle & Krause, 2021) by activating the mathematics register cross-modally, and his gesture providing a (situatively) new, symbolic perspective on the example he started in Farsi. For Ebi, on the other hand, we can observe a switch between the different gestures in an instance of hybrid language practices for his explanation, bearing the potential to scaffold students' meaning making by linking Farsi-vernacular meaning, English-mathematic and semiotic registers bimodally.

Conclusions and Discussion

The goal of this paper was to explore how the analysis of gestures related to hybrid language practices can enhance our understanding of bilinguals' linguistic and cognitive resources in mathematical talk. For this, relevant concepts and literature both from bilingualism in education and the study of gestures as tool to access tacit conceptualisation, as well as cross-linguistic and cross-cultural differences in gesture use, have been synthesized to a framework that considers gestures as intrinsically linked to the linguistic repertoire available for mathematical talk and meaning making.

An application of this framework in two case studies in the context of 'power functions', carried out with two bilingual speakers of Farsi and English—a student and a teacher—illustrated how the analysis of gestures could reveal tacit language-related conceptualizations, in this case linked to the vernacular use of the Farsi mathematics terminology. Furthermore, linking it to the hybrid language practices suggested potential functions of the gestures, in which they might influence the coordination of the two languages in and for making mathematical meaning.

Potential Practical Implications in Multilingual Mathematical Contexts

Gestures are an integral part of communication and part of the resources bi/multilingual students and teachers can draw upon when negotiating mathematical meaning in the mathematics classroom (Farsani, 2015b). The framework tentatively developed in this paper sheds a new perspective on this resource as a source for meaning making by linking it to hybrid language practices in which cultural, linguistic, and representational factors interact and influence the potential for meaning available for the speaker/gesturer. Considering this, teachers can attain a more profound understanding of students' tacit conceptualization of a mathematical idea by observing students' gestures. This also concerns a more comprehensive perspective on a potential additional difficulty for multilingual students: While terminologies with multiple meanings across different contexts (mathematical versus social vernacular) can already be ambiguous and problematic for many students (Pirie, 1991), vernacular meaning of mathematics terminologies in different languages might relate differently to cultural identity. Being more than just translations, understanding bilinguals' hybrid language practices needs to consider more than words to provide an embodied-imagisitic perspective on the meaning of mathematics terminologies as they might activate them against their linguistic and cultural background. An embodied

perspective, therefore, provides a more holistic framework for understanding meaning making in multilingual settings.

We furthermore see a seed for a potential teaching strategy in Ebi's bimodal switch from the Farsi mathematics register to the English one, in which he combines gesture use and hybrid language practices to link mathematical meaning across languages. Elaborating on this is, however, beyond the scope of this paper and left to future research (see also Krause & Farsani, 2021; Krause, 2018 for the Deaf bimodal bilingual classroom).

Implications for Research in Bilingual Mathematics Education

This contribution makes a step towards integrating gesture analysis for investigating bilinguals' use of language in mathematical talk and meaning making. While it frames the conceptual background in a general way, the case studies show a very specific aspect of the potential this might offer. In these specific cases, gesture and speech reflect the background of an understanding of a mathematical term as embedded in linguistic and cultural connections associated within the particular language. It provides a gesture perspective on Prediger and colleagues' (2019) *bilingual complementary mode* and *bilingual connection mode.* What we see in our analyses seems to show how gesture and speech both reflect the bilingual complementarity mode as the embodied-imagistic and the verbalised-linguistic dimensions of the meaning of 'power'/ 'tavan' do not mix across languages. We however also suggest that the gesture perspective might give a better understanding of a link between bilingual complementarity mode and bilingual connection mode: The bilingual connection mode can be observed when Ali combines both languages in forming the final expression of his example, 'tavan equations'. While he explicitly uses 'tavan' in the Farsi mathematics register, his gesture linked to the same concept in the English mathematics register might help connect his Farsi-related conceptualisation with his English-related conceptualisation. For Ebi, we claim to see a glimpse of a teaching strategy—conscious or not—in him merging his gestures going from Farsi to English, performing the '*power*'-gesture as emerging from the final position of the '*tavan*'-gesture and sharing the common gestural feature of the left hand.

It furthermore suggests that the vernacular meaning of the term used in the Farsi mathematics register is culturally linguistically associated also in its usage in the mathematics register, supporting what has been reported by Edmonds-Wathen et al. (2019). With this, the gesture analysis also reveals how switching from Farsi to English seems to mean to switch from a vernacular, culturally accentuated perspective on the mathematical concept talked about to a conventionalized, symbolic one, more strongly integrated in the mathematically cultural norms. With the mathematics terminology strongly linked to its vernacular meaning against the cultural background of the speaker, arriving at a bilingual connection mode might not only mean connecting languages and conceptualisations, but in some cases also cultural identities and conceptualisations.

The case study gives thus an example for how gesture analysis within the wider integration of hybrid language practices and acknowledging ways of thinking that are related to the use of different languages can provide further insights into bilinguals' conceptual understanding and meaning making of mathematics.

The application of the framework for gesture analysis in the case studies also reflects back on the framework itself and allows us to refine it. Figure 2 presented the interpretation of bilinguals' gestures' form-meaning relationships against the background of the three interrelated components of language, culture, and registers coordinated by bilingual students in mathematical talk. The gestures appeared to not only be influenced by this background in their concrete formation, but to influence the manifestation of the components and their coordination with respect to mathematical meaning making, which then revealed itself also in language and the students' use of and reference to registers. We therefore suggest a representation of the model that takes this interaction into account and elevates gesture as its own

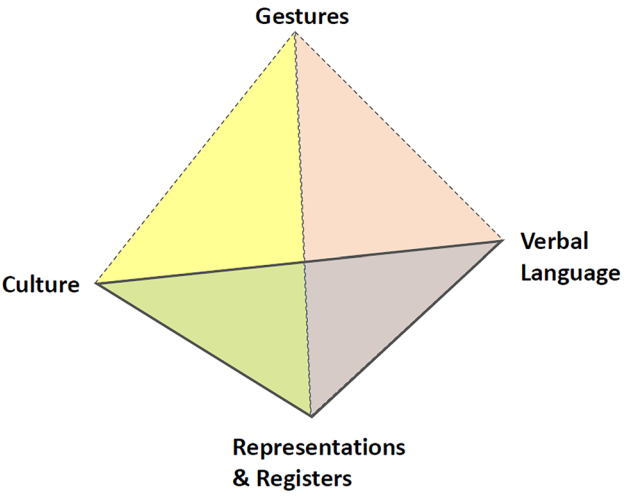


Fig. 7 Model representing gestures as situated in the interaction with language, culture, and registers in bilinguals' mathematical meaning making

additional dimension, links gesture use to each component and thereby, to the face of the resulting tetrahedron as it is spanned by the situated consideration of culture, language, and registers (Fig. 7). We furthermore suggest a potential elaboration of this model to better understand the relationships among the vertices, edges, and faces of this tetrahedron concerning hybrid language practices in bilinguals' mathematics thinking and learning.

Furthermore, this might also be linked with languages that are not spoken but signed, adding to the potential of gestures and (sign) languages in the inclusive mathematics classroom (Krause, 2019; Krause & Wille, 2021).

Final Remarks

Our framework foregrounds ways of integrating gestures as linked to hybrid language practices for mathematical meaning making and thereby provides a general entry point for future research on implementing gesture analysis to better understand bilinguals' cognitive and linguistic (re)sources for mathematical thinking and learning. While the presented case study is very limited in its context, scope, and results, it however provides an idea of how considering bilinguals' gestures can enrich our understanding of their mathematics thinking and learning. This should now be explored more systematically across a variety of multilingual, multicultural, and mathematical contexts, also considering different settings, such as classroom interaction. This way, we can better understand multilinguals' linguistic and cognitive resources for mathematics meaning making and learning and how we can use them to build effective instructional practice beyond words.

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Declarations

Conflict of Interest The authors declare no competing interests.

References

Alibali, M. W., Church, R. B., Kita, S., & Hostetter, A. B. (2014). Embodied knowledge in the development of conservation of quantity: Evidence from gesture. In L. Edwards, F. Ferrara, & D. Moore-Russo (Eds.), *Emerging perspective on gesture and embodiment in mathematics* (pp. 27–50). Information Age Publishing.

Arthur, J. (1996). Code switching and collusion: classroom interaction in Botswana primary schools. *Linguistics & Education, 8*(1), 17–33. https://doi.org/10.1016/S0898-5898(96)90004-2

Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics, 70*(2), 97–109. https://doi.org/10.1007/s10649-008-9163-z

Austin, J. L., & Howson, A. G. (1979). Language and Mathematical Education. *Educational Studies in Mathematics, 10*, 161–197. https://doi.org/10.1007/BF00230986

Barton, B. (2008). *The language of mathematics – Telling mathematical tales*. Springer. https://doi.org/10.1007/978-0-387-72859-9

Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *Journal of Mathematical Behavior, 50*, 155–168. https://doi.org/10.1016/j.jmathb.2018.02.007

Barwell, R. (Eds.) (2009). *Multilingualism in mathematics classrooms: Global perspectives.* Multilingual Matters. https://doi.org/10.21832/9781847692061

Barwell, R. (2008). Discourse, mathematics and mathematics education. In M. Martin-Jones, A. M. de Mejia, & N. H. Hornberger (Eds.), *Encyclopedia of language and education, 2nd edition, Volume 3: Discourse and education* (pp. 317–328). Springer. https://doi.org/10.1007/978-0-387-30424-3_81

Barwell, R. (2014). Centripetal and centrifugal language forces in one elementary school second language mathematics classroom. *ZDM Mathematics Education, 46*(6), 911–922. https://doi.org/10.1007/s11858-014-0611-1

Barwell, R. (2015). Linguistic stratification in a multilingual mathematics classroom. In K. Krainer, & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp.1333–1339). ERME. https://hal.archives-ouvertes.fr/hal-01287658

Barwell, R., Barton, B. & Setati, M. (2007). Multilingual issues in mathematics education: introduction. *Educational Studies in Mathematics, 64*(2), 113–119. https://doi.org/10.1007/s10649-006-9065-x

Blackledge, A., & Creese, A. (2010). *Multilingualism: A Critical Perspective*. Continuum.

Blackledge, A., & Creese, A. (2014). *Heteroglossia as practice and pedagogy*. Springer. https://doi.org/10.1007/978-94-007-7856-6

Boroditsky, L. (2011). How Language Shapes Thought. *Scientific American*, *304*, 62–65.

Bruner, J. S. (1966). *Toward a theory of instruction*. Belkapp Press. https://doi.org/10.1177/019263656605030929

Castellón, V. C., & Enyedy, N. (2006). Teacher's speech and gesture as a communicative and strategic tool to convey and discuss mathematical concepts in a bilingual Algebra classroom. Paper presented at the *Meeting of the American Educational Research Association. San Francisco*.

Clarkson, P.C., & Galbraith, P. (1992). Bilingualism and mathematics learning: Another perspective. *Journal for Research in Mathematics Education, 23*(1), 34–44. https://doi.org/10.2307/749162

Creese, A. Bhatt, A., Bhojani, N., & Martin, P. (2008). Multicultural, heritage and learner identities in complementary schools. *Language and Education*, *20*, 23–43. https://doi.org/10.1080/09500780608668708

Domínguez, H. (2005). Bilingual students' articulation and gesticulation of mathematical knowledge during problem solving. *Bilingual Research Journal, 29*, 269–293. https://doi.org/10.1080/15235882.2005.10162836

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*, 103–131. https://doi.org/10.1007/s10649-006-0400-z

Edmonds-Wathen, C., Owens, K., & Bino, V. (2019). Identifying vernacular language to use in mathematics teaching. *Language and Education, 33*(1), 1–17. https://doi.org/10.1080/09500782.2018.1488863

Edwards, L. D. (2009). Gestures and conceptual integration in mathematical talk. *Educational Studies in Mathematics, 70*(2), 127–141. https://doi.org/10.1007/s10649-008-9124-6

Edwards, L.D., Moore-Russo, D., & Ferrara, F. (Eds) (2014). *Emerging perspectives on gesture and embodiment in mathematics*. Information Age Publishing.

Ellerton, N. F., & Clements, M. K. (1991) *Mathematics in language: A review of language factors in mathematics learning*. Deakin University Press. https://doi.org/10.2307/1169995

Farsani, D. (2015a). *Making multi-modal mathematical meaning in multilingual classrooms.* Unpublished PhD thesis, University of Birmingham.

Farsani, D. (2015b). Deictic gestures as amplifiers in conveying aspects of mathematics register. *Proceedings of the 9th Conference of European Society for Research in Mathematics Education* (pp. 1382–1386). Prague, Czech: ERME. https://hal.archives-ouvertes.fr/hal-01287668

Farsani, D. (2016). Complementary functions of learning mathematics in complementary schools. In P. S. Clarkson, & A. Halai (Eds.), *Teaching & learning mathematics in multilingual classrooms: Issues for policy, practice and teacher education* (pp. 227–247). Sense Publishers. https://doi.org/10.1007/978-94-6300-229-5_15

Farsani, D., Lange, T., & Meaney, T. (2022). Gestures, Systemic Functional Grammar and Mathematics Education. *Mind, Culture and Activity*, 1-21.https://doi.org/10.1080/10749039.2022.2060260

Farrugia, M. T. (2007). *Medium and message: The use and development of an English Mathematics register in two Maltese primary classrooms*. Unpublished doctoral dissertation, University of Birmingham.

García, O., & Wei, L. (2014). *Translanguaging. Language, bilingualism, and education.* Palgrave MacMillan.

Goldin-Meadow, S. (2003). *Hearing gesture: How our hands help us think*. University of Chicago Press.

Goldin-Meadow, S., Alibali, M.W., & Church, R.B. (1993). Transitions in concept acquisition: Using the hand to read the mind. *Psychological Review, 100*(2), 279–297. https://doi.org/10.1037/0033-295x.100.2.279

Gorgorió, N., & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics, 47*(1), 7–33. https://doi.org/10.1023/A:1017980828943

Grosjean, F. (2001). The bilingual's language modes. In J. Nicol (Ed.), *One mind, two languages: Bilingual language processing* (pp. 1–22). Blackwell.

Grosjean, F. (2010). Bilingualism, biculturalism, and deafness. *International Journal of Bilingual Education and Bilingualism, 13*(2), 133–45. https://doi.org/10.1080/13670050903474051

Gullberg, M. (2012). Bilingualism and gesture. In T.K. Bhatia, & W.C. Ritchie (Eds.), *The handbook of bilingualism and multilingualism* (pp. 417–437). Blackwell Publishing Ltd. https://doi.org/10.1002/9781118332382.ch17

Gumperz, J. J., & Hymes, D. H. (1972). *Directions in sociolinguistics: The ethnography of communication*. Holt, Rinehart & Winston.

Gutiérrez, K. D., Baquedano-López, P., & Tejeda, C. (1999). Rethinking diversity: Hybridity and hybrid language practices in the third space. *Mind, Culture, and Activity, 6*(4), 286–303. https://doi.org/10.1080/10749039909524733

Hall, R., & Nemirovsky, R. (2012). Modalities of body engagement in mathematical activity and learning. *Journal of the Learning Sciences, 21*, 207–215. https://doi.org/10.1080/10508406.2011.611447

Halliday, M. (1978). *Language as Social Semiotic. The social interpretation of language and meaning.* Edward Arnold Ltd.

Halliday, M. A. K. (1975) *Learning how to mean: explorations in the development of language*. Edward Arnold, London.

Healy, L., & Fernandes, S. H. A. A. (2011). The role of gestures in the mathematical practices of those who do not see with their eyes. *Educational Studies in Mathematics, 77*(3), 157–174. https://doi.org/10.1007/s10649-010-9290-1

Hostetter, A. B., & Alibali, M. W. (2018). Gesture as simulated action: Revisiting the framework. *Psychonomic Bulletin & Review, 26*, 721–752. https://doi.org/10.3758/s13423-018-1548-0

Jones, D. V., & Martin-Jones, M. (2004). Bilingual education and language revitalization in Wales: Past achievements and current issues. In J. Tollefson, & A. Tsui (Eds.), *Medium of instruction policies: Which agenda? Whose agenda?* (pp. 43–70). Lawrence Erlbaum Associates.

Jørgensen, J. N. (2008). Polylingual languaging around and among children and adolescents. *International Journal of Multilingualism, 5*(3), 161–176. https://doi.org/10.1080/14790710802387562

Kendon, A. (1980). Gesticulation and speech: Two aspects of the process of utterance. In M. R. Key (Ed.), *The relation between verbal and nonverbal communication* (pp. 207–227). The Hague Mouton.

Kita, S. (2009). Cross-cultural variation of speech-accompanying gesture: A review. *Language and Cognitive Processes, 24*(2), 145–167. https://doi.org/10.1080/01690960802586188

Kita, S., Alibali, M. W., & Chu, M. (2017). How do gestures influence thinking and speaking? The gesture-for-conceptualization hypothesis. *Psychological Review, 124*(3), 245–266. https://psycnet.apa.org/doi/https://doi.org/10.1037/rev0000059

Krause, C. M. (2016). *The mathematics in our hands: How gestures contribute to constructing mathematical knowledge.* Springer Spektrum. https://doi.org/10.1007/978-3-658-11948-5

Krause, C.M. (2018). Embodied Geometry: Signs and gestures used in the deaf mathematics classroom – the case of symmetry. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.), *Mathematical discourse*

that breaks barriers and creates space for marginalized learners (pp. 171–193). Sense. http://dx.doi.org/https://doi.org/10.1163/9789004378735_009

Krause, C. M. (2019). What you see is what you get? – Sign language in the mathematics classroom. *Journal for Research in Mathematics Education, 50*(1), 84–97. https://doi.org/10.5951/jresematheduc.50.1.0084

Krause, C.M., & Farsani, D. (2021). Gestures and code-switching in mathematics instruction – an exploratory case study. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education, Vol. 3* (pp. 144–153). PME.

Krause, C. M., & Salle, A. (2019). Towards cognitive functions of gestures—a case of mathematics. In M. Graven, H. Venkat, A. Essien & P. Vale (Hrsg.), *Proceedings of the 43rd Conference of the International Group for the Psychology in Mathematics Education* (Bd. 2, S. 496–503). Pretoria: PME.

Krause, C.M. & Wille, A. M. (2021). Sign language in light of mathematics education: an exploration within semiotic and embodiment theories of learning mathematics. *American Annals of the Deaf, 166*(3), 358-383.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic Books.

Leung, F. K. S. (2017). Making sense of mathematics achievement in East Asia: Does culture really matter? In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 201–218). Springer. https://doi.org/10.1007/978-3-319-62597-3_13

Lucy, J. A. (1996). The scope of linguistic relativity: An analysis and review of empirical research. In J. J. Gumberz, & S. C. Levinson (Eds.), *Rethinking linguistic relativity* (pp. 37–69). Cambridge University Press.

MacSwan, J. (2017). A multilingual perspective on translanguaging. *American Educational Research Journal, 65*(1), 167–201. https://doi.org/10.3102/0002831216683935

Maier, H., & Schweiger, F. (1999). *Mathematik und Sprache. Zum Verstehen und Verwenden von Fachsprache im Unterricht [Mathematics and language. On understanding and use of technical terms in the classroom]*. öbv, hpt.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: an overview of research in the field. *ZDM Mathematics Education 46*, 843–853. https://doi.org/10.1007/s11858-014-0624-9

Morris, N. (2021). Learning probability in the Kingdom of Tonga: the influence of language and culture. *Educational Studies in Mathematics, 107,* 111–134. https://doi.org/10.1007/s10649-020-10022-z

Moschkovich, J. N. (2007). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*(2), 121–144. https://doi.org/10.1007/s10649-005-9005-1

Moschkovich, J. (2019). Codeswitching and mathematics learners. How hybrid language practices provide resources for student participation in mathematical practices. In J. McSwan, & C. J. Faltis (Eds.), *Codeswitching in the classroom: Critical perspectives on teaching, learning, policy, and ideology*. Routledge.

McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. University of Chicago Press.

McNeill, D., & Duncan, S. (2000). Growth Points in Thinking-for-Speaking. In D. McNeill (Ed.), *Language and gesture* (pp. 141–161). Cambridge University Press. https://doi.org/10.1017/CBO9780511620850.010

Miura, I. T., Okamoto, Y., Kim, C. C., Chang, C-M., Steere, M., & Fayol, M. (1994). Comparisons of children's cognitive representation of number: China, France, Japan, Korea, Sweden, and the United States. *International Journal of Behavioral Development, 17*, 401–411. https://doi.org/10.1177/016502549401700301

Nemirovsky, R. (2003). Three conjectures concerning the relationship between body activity and understanding mathematics. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 105–109). PME.

Ng, O. L. (2016). The interplay between language, gestures, dragging and diagrams in bilingual learners' mathematical communications. *Educational Studies in Mathematics, 91*(3), 307–326. https://doi.org/10.1007/s10649-015-9652-9

Ngan Ng, S. S., & Rao, N. (2010). Chinese number words, culture, and mathematics learning. *Review of Educational Research, 80*(2), 180–206. https://doi.org/10.3102/0034654310364764

Otsuji, E., & Pennycook, A. (2011). Social inclusion and metrolingual practices. *International Journal of Bilingual Education and Bilingualism*, *14*, 413–426. https://doi.org/10.1080/13670050.2011.573065

Parvanehnezhad, Z., & Clarkson, P. C. (2008). Iranian bilingual students' reported use of language switching when doing mathematics. *Mathematics Education Research Journal*, *20*(1), 52–81. https://doi.org/10.1007/BF03217469

Pirie, S. (1991). Peer discussion in the context of mathematical problem solving. In K. Durkin, & B. Shire (Eds.), *Language in mathematical education* (pp. 143–161). Open University Press.

Pimm, D. (1987). *Speaking mathematically: Communication in the mathematics classroom.* Routledge.

Poisard, C. Ni Riordain, M., & Le Pipec, E. (2015). Mathematics education in bilingual contexts: Irish-English, Breton-French. *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education*. ERME. https://hal.archives-ouvertes.fr/hal-01287778

Planas, N. (2018). Language as resource: a key notion for understanding the complexity of mathematics learning. *Educational Studies in Mathematics 98*, 215–229. https://doi.org/10.1007/s10649-018-9810-y

Planas, N. (2019). Transition zones in mathematics education research for the development of language as resource. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology in Mathematics Education* (Vol. 1, pp. 17–32). PME.

Prediger, S., & Wessel, L. (2013). Fostering German-language learners' constructions of meanings for fractions—design and effects of a language-and mathematics-integrated intervention. *Mathematics Education Research Journal, 25*(3), 435–456. https://doi.org/10.1007/s13394-013-0079-2

Prediger, S., Clarkson, P., & Bose, A. (2016). Purposefully relating multilingual registers: Building theory and teaching strategies for bilingual learners based on an integration of three traditions. In R. Barwell, P. Clarkson, & A. Halai (Eds.), *Mathematics education and language diversity* (pp. 193–215). Springer. https://doi.org/10.1007/978-3-319-14511-2_11

Prediger, S., Kuzu, T., Schüler-Meyer, A., & Wagner, J. (2019). One mind, two languages – separate conceptualisations? A case study of students' bilingual modes for dealing with language-related conceptualisations of fractions. *Research in Mathematics Education, 21*(2), 188–207. https://doi.org/10.1080/14794802.2019.1602561

Rasmussen, C., Stephan, M., & Allen, K. (2004). Classroom mathematical practices and gesturing. *Journal of Mathematical Behavior, 23*, 301–323. https://doi.org/10.1016/j.jmathb.2004.06.003

Robutti, O., Sabena, C., Krause, C., Soldano, C., & Arzarello, F. (2022). Gestures in mathematics thinking and learning. In M. Danesi (Ed.), *Handbook of cognitive mathematics*. Springer, Cham. https://doi.org/10.1007/978-3-030-44982-7_8-1

Salle, A., & Krause, C. M. (2021). Kognitive Funktionen von Gesten beim mathematischen Arbeiten [Cognitive functions of gestures when working mathematically]. *Journal für Mathematikdidaktik, 42*, 123-158. https://doi.org/10.1007/s13138-020-00169-w

Setati, M., & Adler, J. (2000). Between languages and discourses: Language practices in primary mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*(3), 243 –269. https://doi.org/10.1023/A:1011996002062

Shein, P. P. (2012). Seeing with two eyes: A teacher's use of gestures in questioning and revoicing to engage English language learners in the repair of mathematical errors. *Journal for Research in Mathematics Education, 43*(2), 182–222. https://doi.org/10.5951/jresematheduc.43.2.0182

Schüler-Meyer, A., Prediger, S., Kuzu, T., Wessel, L., & Redder, A. (2019). Is formal language proficiency in the home language required to profit from a bilingual teaching intervention in mathematics? A mixed methods study on fostering multilingual students' conceptual understanding. *International Journal of Science and Mathematics Education, 17*(2), 317–399. https://doi.org/10.1007/s10763-017-9857-8

Sparhawk, C. M. P. (1981). Contrastive-identificational features of Persian gesture. In Kendon, A. (Ed.), *Nonverbal communication, interaction, and gesture* (pp. 421–458). The Hague Mouton. https://doi.org/10.1515/semi.1978.24.1-2.49

Varela, F. J., Thompson, E. T., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. MIT Press.

Zurina, H., & Williams, J. (2011). Gesturing for oneself. *Educational Studies in Mathematics, 77*(3), 175–188. https://doi.org/10.1007/s10649-010-9294-x